# **COURSE 12: LINEAR ALGEBRA**

Theory

Credits: 4

5 hrs/week

#### **Course Outcomes**

After successful completion of this course, the student will be able to

- 1. understand the concepts of vector spaces, subspaces
- 2. understand the concepts of basis, dimension and their properties
- 3. understand the concept of linear transformation and its properties
- 4. apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods
- 5. learn the properties of inner product spaces and determine orthogonality in inner product spaces.

### **Course Content**

# UNIT – I

#### **Vector Spaces-I**

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - addition and scalar multiplication of Vectors - internal and external composition - Null space - Vector subspaces - Algebra of subspaces - Linear Sum of two subspaces - linear combination of Vectors- Linear span Linear independence and Linear dependence of Vectors.

### UNIT –II

# **Vector Spaces-II**

Basis of Vector space - Finite dimensional Vector spaces - basis extension - co-ordinates- Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space.

# UNIT –III

# **Linear Transformations**

Linear transformations - linear operators- Properties of L.T- sum and product of L.Ts - Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformations - Rank- Nullity Theorem.

#### UNIT -IV

#### Matrices

Characteristic equation - Characteristic Values - Characteristic vectors of a square matrix - Cayley Hamilton Theorem – problems on Cayley Hamilton Theorem.

#### UNIT -V

#### **Inner product space**

Inner product spaces- Euclidean and unitary spaces- Norm or length of a Vector- Schwartz inequality-Triangle Inequality- Parallelogram law- Orthogonality- Orthonormal set- Problems on Gram– Schmidt orthogonalisation process - Bessel's inequality.

#### Activities :

Seminar/ Quiz/ Assignments/Applications of Linear Algebra in real life problems\ Problem Solving.

# **Text Books**

- 1.Linear Algebra by J.N. Sharma and A.R. Vasishtha, published by Krishna Prakashan Media (P) Ltd.
- 2.Matrices by A.R.Vasishtha and A.K.Vasishtha published by Krishna Prakashan Media (P) Ltd.

# **Reference Books**

- 1. Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt. Ltd. 4<sup>th</sup> Edition, 2007
- 2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson education low priced edition), New Delhi.
- 3. Matrices by Shanti Narayana, published by S.Chand Publications \*\*\*\*\*

# **COURSE 13: VECTOR CALCULUS**

Theory Credits: 4 5 hrs/week

### **Course Outcomes**

Students after success ful completion of the course will be able to

- 1. Learnmultipleintegrals asanaturalextensionofdefiniteintegraltoafunctionoftwovariables inthecaseofdoubleintegral/threevariables inthecaseoftripleintegral.
- 2. Learnapplicationsintermsoffindingsurfaceareabydoubleintegralandvolumebytripleintegral
- 3. Determinethegradient, divergence and curlof avector and vector identities.
- 4. Evaluateline, surface and volume integrals.
- 5. understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem)

# **Course Content**

# Unit–1

# MultipleIntegrals-I

Introduction -Doubleintegrals -Evaluationofdoubleintegrals –Propertiesofdouble integrals – Regionofintegration -doubleintegrationinPolarCo-ordinates – Changeofvariablesindoubleintegrals -changeoforderofintegration.

# Unit–2

# Multipleintegrals-II

Tripleintegral -regionofintegration -changeofvariables -Plane areasbydoubleintegrals - surfaceareabydoubleintegral -Volumeasadoubleintegral, volumeasatripleintegral.

# Unit-3

# Vectordifferentiation

Vectordifferentiation –ordinary - derivativesofvectors – Differentiability –Gradient –Divergence - Curloperators - Formulaeinvolvingtheseparators.

# Unit-4

#### Vectorintegration

Line Integralswithexamples - Surface Integralwithexamples - Volumeintegralwithexamples.

# Unit–5

# Vectorintegrationapplications

Gausstheorem and applications of Gausstheorem - Green's theorem in plane and applications of Green's theorem - Stokes's theorem and applications of Stokes theorem.

# Activities

Seminar/ Quiz/ Assignments/ Applications of Vector calculus to Real life Problems /Problem Solving Sessions.

# **Text Book**

A text Book of Higher Engineering Mathematics by B.S.Grawal, Khanna Publishers, 43<sup>rd</sup> Edition ReferenceBooks

- Vector Calculus by P.C.Matthews, Springer Verlag publications.
  Vector Analysis by Murray Spiegel, Schaum Publishing Company, NewYork

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### **COURSE 14: FUNCTIONS OF A COMPLEX VARIABLE**

Theory

Credits: 4

5 hrs/week

#### **Course Outcomes**

After successful completion of this course, the student will be able to

- 1. determine a Bilinear transformation under given condition
- 2. know about continuity, compactness and connectedness of sets in complex plane
- 3. know the necessary condition and sufficient condition for f(z) to be analytic
- 4. know about the inverse of an analytic function
- 5. know about the convergence of sequences and the necessary & sufficient condition for a sequence to be convergent
- 6. know the power series expansion of elementary functions

#### **Course Content**

#### Unit – 1

#### **Bilinear Transformations**

Extended Complex Plane – Resultant and Inverse of a bilinear transformation – The linear group – Geometrical significance of the transformation. Angle preserving property of Bilinear Transformation– Determination of Bilinear transformations under given condition, some special bilinear transformations.

#### **Unit** – 2

#### **Topological Considerations**

Neighbourhood of a point – Interior, exterior and frontier points of a set, open and closed sets. Connected sets, Domains and continua - a theorem on Nests of closed Rectangular domains- Bolzano Weierstrass theorem- Hein-Borel theorem. Limits - algebraic operations with limits – continuity and uniform continuity – compactness – connectedness - Jordan curve theorem - connectedness of line segments and polygonal lines. Branch line and Branch point - Characterisation of open connected sets by polygonal lines.

#### Unit – 3

#### **Analytic functions**

Differentiable functions of a complex variable - Geometrical representation of a variable - Analytic function- Elementary rules and chain rule - Derivatives of polynomials and rational functions - The necessary condition and sufficient condition for f(z) to be analytic - Analytic functions in a Domain – Derivative of w in polar form - Construction of f(z).

#### **Unit** – 4

### Inverse of an analytic function and infinite series

The inverse of an analytic function – neighbourhood preserving mappings - Domain preserving and angle preserving property of analytic mappings.

Convergent sequences, necessary and sufficient condition for a sequence to be convergent, Cauchy sequence, Convergence of infinite series. Cauchy general principle of convergence for a series. Absolute convergence of a series. Abel's and Dirichilet's tests. Rearrangement of series, product of series.

# Unit – 5

### **Power Series**

Power series - exponential, trigonometric and hyperbolic functions - zeros of sin z, cos z - periods of sin z, cos z, E(z) - A law of logarithms - Analytic character of log z - generalized  $a^b$  - Analytic character of  $z^n$  - Cos<sup>-1</sup> z, Sin<sup>-1</sup> z and derivatives of Cos<sup>-1</sup> z, Sin<sup>-1</sup> z.

### Activities

Seminar/ Quiz/ Assignments/ Applications of Functions of complex variables to Real life Problem /Problem Solving Sessions.

### **Text Book**

Theory of Functions of a Complex variable by Shanti Narayan &Dr. P. K. Mittal, S. Chand &Company Ltd.

### **Reference Books**

- 1. Theory of Functions of a Complex Variable by A. I. Markushevich, Second Edition, AMS Chelsea Publishing
- 2. Theory And Applications by M. S. Kasara, Complex Variables, 2nd Edition, Prentice Hall India Learning Private Limited

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### **COURSE 14: ADVANCED NUMERICAL METHODS**

Theory Credits: 4 5 hrs/week

#### **Course Outcomes**

After successful completion of this course, the student will be able to

- 1. find derivatives using various difference formulae
- 2. understand the process of Numerical Integration
- 3. solveSimultaneous Linear systems of Equations

4. understand Iterative methods

5. find Numerical Solution of Ordinary Differential Equations

### **Course Content**

### UNIT – I

# **Numerical Differentiation**

Derivatives using Newton's forward difference formula - Newton's backward difference formula-Derivatives using central difference formula - Stirling's interpolation formula - Newton's divided difference formula.

# UNIT – II

### **Numerical Integration**

General quadrature formula on errors - Trapezoidal rule – Simpson's 1/3 rule - Simpson's 3/8 rule-Weddle's rule - Euler-Maclaurin formula of summation and quadrature - The Euler transformation.

### $\mathbf{UNIT}-\mathbf{III}$

### Solution of Simultaneous Linear systems of Equations – I

Solution of linear systems - Direct Methods - Matrix inversion method – Gaussian elimination method-Gauss Jordan Method.

# $\mathbf{UNIT} - \mathbf{IV}$

# Solution of Simultaneous Linear systems of Equations – II

Method of factorization - solution of Tridiagonal systems - Iterative methods - Jacobi's method - Gauss - Siedal method.

# $\mathbf{UNIT} - \mathbf{V}$

# Numerical Solution of Ordinary Differential Equations

Introduction – solution of Taylor's series – Picard's method of successive approximations – Euler's method – Modified Euler's method – Runge-Kutta methods.

#### Activities

Seminar/ Quiz/ Assignments/ Applications of Numerical methods to Real life Problem /Problem Solving Sessions.

#### **Text Book**

Numerical Analysis by G. Shanker Rao, New Age International Publications

# **Reference Books**

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, Pearson Publications.

2. Numerical Methods for Scientific and Engineering Computation by M. K. Jain, S. R. K. Iyengar and

R. K. Jain, New Age International Publishers.

### **COURSE 15: NUMBER THEORY**

5 hrs/week

Theory	Credits: 4	

#### **Learning Outcomes**

After successful completion of the course, students will be able to

1. understand the fundamental theorem of arithmetic

2. understand Mobius function, Euler quotient function, The Mangoldt function, Liouville's function, The divisor functions and the generalized convolutions.

3.understand Euler's summation formula, application to the distribution of lattice points and the applications to  $\mu$  (n) and  $\Lambda$  (n)

4. understand the concepts of congruencies, residue classes and complete residues systems.

5. Comprehend the concept of quadratic residues mod p and quadratic non residues mod p.

# UNIT-I

# The Fundament Theorem of Arithmetic

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, The greatest common divisor of more than two numbers

#### **UNIT-II**

#### **Arithmetical Functions And Dirichlet Multiplication**

Introduction- The Mobius function  $\mu(n)$  – The Euler quotient function  $\varphi(n)$  - A relation connecting  $\varphi$  and  $\mu$  - A product formula for  $\varphi(n)$  - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function  $\Lambda(n)$ - multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function  $\lambda(n)$  - The divisor functions  $\sigma_{\alpha}(n)$ 

#### UNIT-III

#### **Averages Of Arithmetical Functions**

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula-Some elementary asymptotic formulas-The average order of d(n)- The average order of the divisor functions  $\sigma_{\alpha}$  (*n*)- The average order of  $\varphi$  (*n*)- An application to the distribution of lattice points visible from the origin- The average order of  $\mu$  (*n*) and  $\Lambda(n)$ -The partial sums of a Dirichlet product-Applications to  $\mu$  (*n*) and  $\Lambda$  (*n*)

### UNIT-IV

#### Congruences

Definition and basic properties of congruences- Residue classes and complete residue systems-Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p. Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem

### UNIT-V

### Quadratic Residues and the Quadratic Reciprocity Law

Quadratic Residues, Legendre's symbol and its properties, Evaluation of (-1/p) and (2/p), Gauss lemma, The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol, Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums, Another proof of the quadratic reciprocity law.

### Activities

Seminar/ Quiz/ Assignments/ Applications of Number theory to Real life Problem /Problem Solving Sessions

### **Text Book**

Introduction to Analytic Number Theory by T.M.Apostol, Springer Verlag-New York, Heidalberg-Berlin-1976.

### **Reference Books**

- 1. Elementary Number Theory by G.A.Jones and J.M.Jones, , Springer
- 2. Elementary Number Theory by David, M. Burton, 2nd Edition UBS Publishers.
- 3. Number Theory by Hardy & Wright, Oxford Univ., Press.
- 4. Elements of the Theory of Numbers by Dence, J. B & Dence T.P, Academic Press

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# **COURSE 15: MATHEMATICAL STATISTICS**

Theory

Credits: 4

5 hrs/week

#### **Course Outcomes**

After completion of the course, student will be able to

- 1. understandthe probability set function and conditional probability
- 2. understand about random variables, discrete and continuous type distributions
- 3. understand the distribution of two random variables and expectation of a random variables
- 4. know binomial and related distributions
- 5. normal distributions and the applications of normal distributions

# **Unit** – 1

# **Probability and Distributions**

Sets – set functions – The probability set function – counting rules – additional properties of probability- conditional probability and independence - simulations

#### **Unit** – 2

### Probability and Distributions continued..

Random Variables - Discrete Random Variables - Continuous Random Variables -Quantiles-Transformations - Mixtures of Discrete and Continuous Type Distributions Expectation of a Random Variable - Computation for an Estimation of the Expected Gain - Some Special Expectations - Important Inequalities

#### **Unit** – 3

### **Multivariate Distributions**

Distributions of Two Random Variables - Marginal Distributions - Expectation – Transformations Bivariate Random Variables - Conditional Distributions and Expectations - Independent Random Variables - The Correlation Coefficient - Extension to Several Random Variables Multivariate Variance-Covariance Matrix- Transformations for Several Random Variables - Linear combinations of Random Variables

#### Unit – 4

### **Some Special Distributions**

The Binomial and Related Distributions - Negative Binomial and Geometric Distributions - multinomial Distribution- Hypergeometric Distribution - The Poisson Distribution - The  $\Gamma$ ,  $\chi$ 2 and  $\beta$  Distributions - The  $\chi$ 2-Distribution - The  $\beta$ -Distribution

#### Unit – 5

#### **Normal Distribution**

The Normal Distribution. - Contaminated Normals - The Multivariate Normal Distribution - Bivariate Normal Distribution - Multivariate - Normal Distribution. General Case- Applications -t- and F-Distribution

#### Activities

Seminar/ Quiz/ Assignments/ Applications of Mathematical statistics to Real life Problem /Problem Solving Sessions.

**Text Book** 

Introduction to Mathematical Statistics by Robert V Hogg, Joseph W MacKeen, Eighth Edition, Allen T Craig, Pearson

### **Reference Books**

1. Fundamentals of Statistics by Goon A.M., Gupta M.K. and Dasgupta B., (2002) Vol. I & II, 8th Edn. The World Press, Kolkata.

2. Fundamentals Of Mathematical Statistics by Gupta, S. C. and Kapoor, V.K. (2008): 4 thEdition (Reprint), Sultan Chand &Sons

3. Mathematical Statistics with Applications by Miller, Irwin and Miller, Marylees(2006) John E.Freund's, (7th Edn.), Pearson Education, Asia.

4. Introduction to the Theory of Statistics by Mood, A.M. Graybill, F.A. and Boes, D.C., (2007), 3<sup>rd</sup>Edn., (Reprint), Tata McGraw-Hill Pub. Co.Ltd.

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